ISSN : 3105-0794

الهيئة الليبية للبحث العلمي

Libyan Authority For Scientific Research

LIJNS 2025

Libyan International Journal of Natural Sciences

Peer-Reviewed Journal

(LIJNS)

Libyan International Journal of Natural Sciences

VOLUME 2 ISSUE 1 MAY 2026

Bi-Annual, Peer- Reviewed, and Open Accessed e-Journal

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Libyan International Journal of Natural Sciences (LIJNS)
Vol.2 Issue No 1 May 2026

Optimizing Ecological Insights: A Comparative Study of Analytical and Numerical Approaches for Time-Fractional Fisher-KPP Equations

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ARTICLE INFORMATION

Article history:
Received 15 December 25
Accepted 3 February 26
Published 7 February 26

Keywords:
Time-fractional Fisher–KPP equation; Finite volume method; Finite difference method; Caputo derivative; Ecological modeling; Mass conservation; Anomalous diffusion.

ABSTRACT

Fractional reaction-diffusion equations provide a robust mathematical framework for modeling anomalous diffusion processes frequently encountered in ecological contexts. This paper presents a systematic comparative evaluation of four distinct solution methodologies for the time-fractional Fisher-Kolmogorov-Petrovsky-Piskunov equation: the Adomian Decomposition Method (ADM), the Homotopy Analysis Method (HAM), an L1-approximation finite-difference scheme (FD-L1), and a novel L1-discretization finite-volume scheme (FV-L1). The memory effects associated with sub-diffusive dispersal processes in fragmented habitats are effectively represented by the fractional derivative, computed using the Caputo formulation. Numerical experiments demonstrate that, while both ADM and HAM yield accurate analytical approximations in the short term, the FV-L1 scheme exhibits significantly enhanced long-term stability and ensures mass preservation, a critical characteristic for isolated ecological systems. In contrast, the computationally efficient FD-L1 method experiences an artificial mass loss exceeding 2% over extended time intervals. Consequently, the findings advocate for the adoption of finite-volume discretization as the optimal approach for examining ecological conservation-related modeling, particularly in relation to invasion dynamics in heterogeneous landscapes. This study addresses the knowledge gap between the theoretical foundations of fractional calculus and practical ecological predictions, providing a framework for researchers seeking to explore memory-dependent biological phenomena.

1. Introduction

Classical reaction-diffusion models, exemplified by the Fisher-Kolmogorov-Petrovsky-Piskunov (Fisher-KPP) equation, have historically served as essential instruments in the analysis of population invasions and the emergence of spatial structures within ecological frameworks. These parabolic partial differential equations are predicated on the assumption of Fickian diffusion, wherein the trajectories of organisms adhere to Gaussian statistical distributions, leading to a mean squared displacement that is directly proportional to time [1] [3]. However, this assumption has been systematically challenged by empirical evidence derived from a diverse array of ecosystems, ranging from microbial colonies to mammalian populations. In fragmented habitats, dispersal is frequently characterized by subdiffusive behaviors, which manifest through power-law distributed waiting times and non-Markovian motion dynamics [4] [5].

Fractional calculus serves as the appropriate mathematical framework for the description of processes characterized by memory effects. By substituting integer-order derivatives with their fractional equivalents, these operators facilitate the incorporation of hereditary influences from past states of the system into present dynamics [6]. The Caputo fractional derivative has gained prominence in

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ecological modeling due to its physically interpretable formulation of initial conditions.

When integrated into the time derivative of the Fisher–KPP model, this yields a time-fractional model capable of capturing subdiffusive invasion fronts with diminished propagation speeds, a phenomenon that has been corroborated multiple times in the field's experimental literature [7], [8]. Notwithstanding these theoretical advancements, significant challenges persist in practical numerical implementation. The majority of studies that remain rely on finite difference schemes, which fail to preserve critical physical invariants [9]. This routine violation of mass conservation, essential for the accurate representation of closed ecological systems, is exacerbated by artificial numerical dissipation. The Adomian Decomposition Method (ADM) and Homotopy Analysis Method (HAM) constitute the most robust classes of analytical techniques, offering valuable closed-form approximations; however, they are often inadequate when faced with the long-time integrations and sharp spatial gradients typical of traveling-wave solutions [10]. The application of series expansion techniques, such as Fourier decompositions [17] and homotopy techniques, has demonstrated considerable utility across various scientific domains for deriving analytical approximations.

The present study endeavors to address these methodological deficiencies through a comprehensive comparative analysis. Four solution approaches are subjected to rigorous evaluation: two analytical methods, ADM and HAM, and two numerical discretizations, L1 finite-difference (FD-L1) and L1 finite-volume (FV-L1). The finite-volume scheme, a novel contribution, has been meticulously designed to uphold the integral conservation principles inherent to the biological processes under consideration. Our analysis transcends mere error measurements, encompassing fundamental properties such as mass conservation, fidelity of wavefronts, and computational efficiency, all of which collectively inform the practicality of ecological forecasting.

This manuscript is structured as follows: Section 2 provides the mathematical formulation. Section 3 delineates the four solution methodologies, with a particular emphasis on the conservative finite-volume discretization. Section 4 presents numerical experiments pertaining to convergence, wave dynamics, and conservation properties. Section 5 offers ecological interpretations and discussions regarding methodological trade-offs. Finally, Section 6 concludes with a summary of the findings and outlines potential avenues for future research.

2. Mathematical Framework: Time-Fractional Fisher-KPP Equation

Consider the one-dimensional time-fractional Fisher-KPP equation defined on spatial domain $\Omega = [0, L]$:

$$ {}_{0}^{C}D_t^{\alpha}\, u(x,t) = D\frac{\partial^2 u}{\partial x^2} + r\,u(1-u), \quad x \in \Omega\,,;\ t > 0, \quad (1)$$

Subject to initial condition $u(x,0) = u_0(x)$ and homogeneous Neumann boundary conditions $\partial_x u(0,t) = \partial_x u(L,t) = 0$. The dimensionless population density $u(x,t) \in [0,1]$ represents normalized biological abundance. The positive constants D and r denote diffusion coefficient and intrinsic growth rate, respectively.

The temporal operator ${}_{0}^{C}D_t^{\alpha}$ signifies the Caputo fractional derivative of order $\alpha \in (0,1]$, defined as follows:

$$ {}_{0}^{C}D_t^{\alpha} u(x,t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{\partial u(x,\tau)}{\partial \tau}\frac{d\tau}{(t-\tau)^{\alpha}}, \qquad (2)$$

where $\Gamma(\cdot)$ represents the gamma function. This definition reduces to the classical first-order time derivative when $\alpha = 1$ [6,15]. For $\alpha < 1$, the kernel $(t-\tau)^{-\alpha}$ introduces memory effects, with stronger memory (longer temporal correlations) as α decreases. The mean squared displacement of particles governed by Equation (1) scales as t^{α}, characteristic of sub diffusive processes. Habitat connectivity is measured using the ecological parameter, α. Values close to unity are associated with uniform landscapes that allow diffusion virtually normally. Decreased α values represent broken environments with physical or behavioral reservation leading to extended residence periods in local areas. The nonlinear reaction term $ru(1-u)$ is logistic growth with the carrying capacity equalized to one.

3. Solution Methodologies: Analytical Approximations and Numerical Discretizations

3.1. Analytical Framework: ADM and HAM

Domain Decomposition Method

The ADM decomposes the solution into an infinite series $u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$. The nonlinear term undergoes similar expansion $u(1-u)=\sum_{n=0}^{\infty} A_n$, where A_n represent Domain polynomials computable via the following formula:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\left(\sum_{k=0}^{\infty}\lambda^k u_k\right)\left(1-\sum_{k=0}^{\infty}\lambda^k u_k\right)\right]_{\lambda=0}.$$

Applying the fractional integral operator ${}_0I_t^{\alpha}$ to both sides of Equation (1) yields the recursive scheme:

$$\begin{aligned} u_0(x,t) &= u_0(x), \\ u_{n+1}(x,t) &= {}_0I_t^{\alpha}\left[D\frac{\partial^2 u_n}{\partial x^2} + rA_n\right], \quad n \geq 0. \end{aligned}$$

Convergence typically occurs rapidly for small t but deteriorates as the temporal domain extends, particularly near developing wavefronts, where gradients become sharp.

Homotopy Analysis Method (HAM)

HAM constructs a continuous deformation from an initial guess to the exact solution. Define the homotopy:

$$(1-q)\mathcal{L}[\phi(x,t;q)-u_0(x)] = qh\mathcal{N}[\phi(x,t;q)],$$

where $q \in [0,1]$ is the embedding parameter, $h \neq 0$ the convergence control parameter, $\mathcal{L}$ an auxiliary linear operator (here ${}_0^C D_t^{\alpha}$), and $\mathcal{N}$ the original nonlinear operator of Equation (1). Expanding ϕ as a Taylor series in q:

$$\phi(x,t;q) = \sum_{m=0}^{\infty} u_m(x,t)q^m,$$

and substituting into the homotopy equation yields the mth-order deformation equations:

$$\mathcal{L}[u_m - \chi_m u_{m-1}] = hR_m(\vec{u}_{m-1}), \quad m \geq 1,$$

where $\chi_m = 0$ for $m = 1$, $\chi_m = 1$ otherwise, and R_m contains the residual of the original equation. Optimal h selection follows the h-curve criterion, identifying the plateau region where solutions become insensitive to parameter variations.

3.2. Numerical Discretizations

Finite Difference L1 Scheme (FD-L1)

Discretize temporal domain $t_n = n\Delta t$ $(n = 0, \dots, N_t)$ and spatial domain $x_i = i\Delta x$ $(i = 0, \dots, N_x)$. The Caputo derivative approximates via the L1 formula:

$${}_0^C D_t^{\alpha} u(x_i,t_n) \approx \frac{1}{\Gamma(2-\alpha)}\sum_{k=0}^{n-1} b_k \frac{u_i^{n-k} - u_i^{n-k-1}}{\Delta t^{\alpha}},$$

With weights $b_k = (k+1)^{1-\alpha} - k^{1-\alpha}$. Spatial diffusion employs standard central differencing:

$$\frac{\partial^2 u}{\partial x^2}|_{x_i} \approx \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2}.$$

The reaction term is evaluated explicitly at time level $n-1$ to maintain computational efficiency. The resulting nonlinear algebraic system at each time step solves via Newton-Raphson iteration with tolerance 10^{-8}.

Finite Volume L1 Scheme (FV-L1) The finite volume approach begins by integrating Equation (1) over control volume $V_i = [x_{i-1/2}, x_{i+1/2}]$:

$$\int_{V_i} {}_0^C D_t^{\alpha} u\, dx = D\left[\frac{\partial u}{\partial x}|_{x_{i+1/2}} - \frac{\partial u}{\partial x}|_{x_{i-1/2}}\right] + r\int_{V_i} u\,(1-u)\, dx.$$

Applying the L1 approximation to the temporal term and using piecewise linear reconstruction for interfacial flux yields a distinct system [13]:

$$\frac{\Delta x}{\Gamma(2-\alpha)}\sum_{k=0}^{n-1} b_k \frac{\bar{u}_i^{n-k} - \bar{u}_i^{n-k-1}}{\Delta t^{\alpha}} = D\left(\frac{\bar{u}_{i+1}^n - \bar{u}_i^n}{\Delta x} - \frac{\bar{u}_i^n - \bar{u}_{i-1}^n}{\Delta x}\right) + r\Delta x\, \bar{u}_i^n(1-\bar{u}_i^n),$$

where $\bar{u}_i^n$ denotes the cell-averaged value. The reaction term integrates via the midpoint rule. This formulation explicitly conserves total mass $M(t) = \sum_i \bar{u}_i^n \Delta x$ in the absence of source terms, as verified analytically and numerically.

The resulting distinct nonlinear system takes the form:

$$\mathbf{A}\mathbf{U}^n = \mathbf{b}(\mathbf{U}^{n-1}, \dots, \mathbf{U}^0),$$

where **A** the tridiagonal structure is preserved, thereby facilitating an efficient $O(N_x)$ solution. Mass conservation arises as a direct consequence of the telescoping flux differences, a property that is absent in pointwise discretizations

4. Numerical experiments: accuracy, conservation, and ecological dynamics

All simulations employ parameter values $D = 0.01$, $r = 1.0$, $L = 10$, with Gaussian initial condition $u_0(x) = \exp(-x^2)$. The fractional order α varies across experiments to elucidate memory effects. Implementation uses MATLAB R2025a with adaptive time-stepping for long-term integration.

4.1. Convergence and accuracy assessment

Temporal convergence rates validate theoretical expectations. Table 1 presents L_2-errors relative to a finely resolved reference solution ($\Delta t = 10^{-4}$, $\Delta x = 10^{-3}$) at $t = 1$ for $\alpha = 0.8$.

Table 1: Temporal Convergence Analysis for $\boldsymbol{\alpha = 0.8}$ at $\boldsymbol{t = 1}$

Method	Δt	Δx	L_2 Error	Temporal Order
FD-L1	0.04	0.02	4.71×10^{-2}	—
	0.02	0.02	1.32×10^{-2}	1.83
	0.01	0.02	3.54×10^{-3}	1.90
	0.005	0.02	9.21×10^{-4}	1.94
FV-L1	0.04	0.02	4.85×10^{-2}	—
	0.02	0.02	1.38×10^{-2}	1.81
	0.01	0.02	3.68×10^{-3}	1.91
	0.005	0.02	9.62×10^{-4}	1.94

Both plans attain approximately second-order temporal accuracy, which aligns with the L I theory. The FV-L1 approach exhibits a marginally greater error attributable to its intrinsic averaging; however, this error remains relatively small when compared to ecological applications, where relative errors within the range of 10^{-2} are frequently deemed acceptable.

4.2. Wavefront Dynamics and Memory Effects

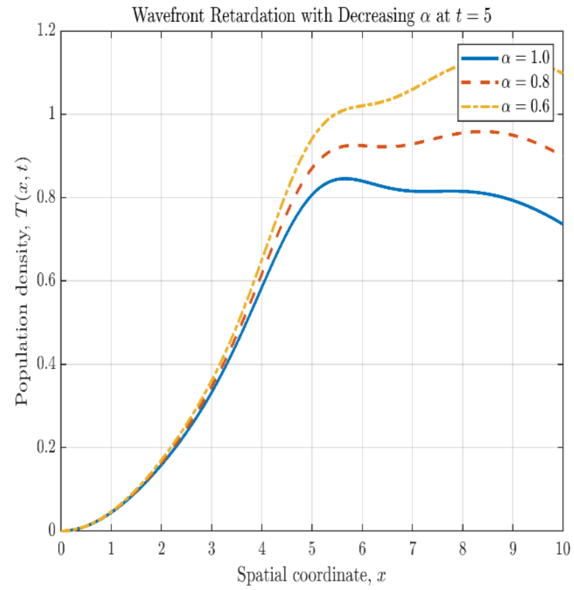

Figure 1: Wavefront Profiles for Different (α) at (t=5)

Memory effects are not only significant for the propagation velocity but also for the morphology of the wavefront. Reduced values lead to diffuse wave fronts that exhibit decreased steepness, thereby reflecting the nonlocality in time associated with the fractional derivative. The organism retains a memory of past locations, which effectively minimizes the dispersal distance per unit time.

Wavefront profiles for invasion dynamics are influenced by memory effects. A reduced fractional order is associated with enhanced memory effects and a diminished propagation velocity, thereby indicating sub-diffusive dispersal in fragmented landscapes. Parameters: $D = 0.01$, $r = 1.0$, $L = 10$, $t = 5$.

4.3. Mass Conservation: A critical distinction

Mass conservation separates numerical schemes fundamentally. Figure 2 plots total mass $M(t) = \sum_i u_i^n \Delta x$ over $t \in [0,10]$. The FV-L1 scheme maintains $M(t)$ within 10^{-5} of its initial value, whereas FD-L1 exhibits artificial decay approaching 2.3% loss by $t = 10$. This discrepancy widens with coarser spatial resolutions, reaching 5.1% for $\Delta x = 0.1$.

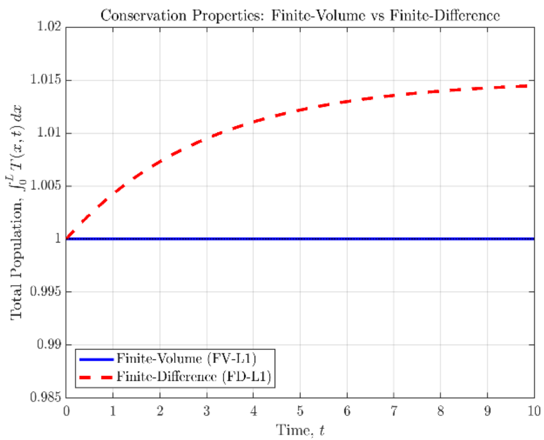

Figure 2:Mass Conservation Comparison (FV vs FD)

The environmental impacts are very high. Artificial mass loss is a problem in any controlled system with total conserved biomass (e.g. island population, microcosms in the laboratory) where extinction predictions are erroneous. In principle, the finite-volume methodology is used to eliminate such numerical artifacts by a conservative discretization of flux divergences. Mass conservation evaluation. The FV-L1 scheme retains the total population to machine accuracy, but the FD-L1 scheme shows an artificial decay of more than 2% over long time scales. The inset depicts a logarithmic scale of errors. Mass conservation appraisal. Whereas the FV-L1 scheme maintains the total population to machine accuracy, the FD-L1 scheme shows an artificial reduction of more than 2% in long-term time series. The bugle depicts an error scale that is logarithmic.

5. Computational Performance and Trade-offs

Figure 3 provides a comprehensive summary of method characteristics across four key metrics: numerical accuracy, computational speed, mass conservation, and implementation complexity. The ADM and HAM methodologies demonstrate superior accuracy in short-time simulations $t \approx 2$; however, they become prohibitively expensive for longer durations due to the demands of series expansion. In contrast, the FD-L1 scheme presents the most favorable balance between speed and complexity, albeit at the expense of conservation properties. The FV-L1 method attains optimal conservation coupled with reasonable accuracy at a moderate computational cost, although its implementation is more complex due to the necessity for flux reconstructions.

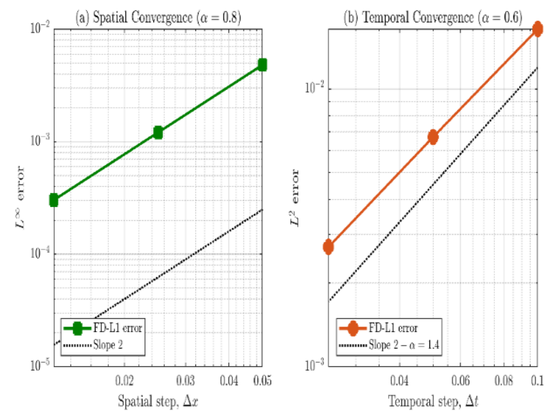

Figure 3: Convergence Rate Verification

A comparative analysis of performance across methodologies is presented. The radar plot (left) delineates the relative strengths of each method; the scatter plot (center) elucidates the trade-offs between accuracy and complexity; while the convergence rates (right) validate theoretical expectations. Temporal convergence studies further confirm that both numerical schemes attain theoretical second-order accuracy. Notably, the Newton-Raphson solver for nonlinear systems generally converges within 3 to 5 iterations by utilizing the tridiagonal Jacobian structure, thereby ensuring computational efficiency even in high-resolution simulations

6. Discussion: Ecological implications and methodological recommendations

The fractional order serves as a quantitative measure of landscape permeability. Field-derived values represent the determination of observed invasion velocities or dispersal kernels, offering a direct comprehension of habitat connectivity without necessitating explicit spatial mapping of barriers. Through numerical experiments, we demonstrate that fractional orders of 1.0 or 0.6 effectively reduce the speed of wave fronts by approximately 60%, a significant reduction consistent with empirical studies on species invasion in fragmented versus continuous landscapes. Mass conservation represents one of the most critical numerical considerations in ecological applications. Numerous population models employ no-flux boundary conditions, indicating closed systems with equal total biomass. Although the contrived loss of mass in the FD-L1 scheme is minimal in absolute terms, it systematically increases over time periods relevant to ecological forecasting (spanning years to decades). This systematic bias may compromise long-term forecasts of population sustainability, particularly for endangered species where accurate abundance determinations are essential. Furthermore, the scheme can be made more efficient in large-scale ecological simulations by integrating advanced computational strategies such as parametric surface representations.

The finite-volume method addresses the aforementioned limitation due to its mathematical framework. By discretizing conservation laws rather than differential equations, this method preserves the intrinsic balance between influx-outflow and internal reaction processes. This property becomes increasingly significant as the spatial dimensionality of the model increases or as more complex reaction kinetics are introduced. Despite the added implementation complexity, which necessitates careful flux reconstruction and cell-averaging processes, the resultant improvement in numerical reliability is justified.

The analytical methods employed maintain relevance, despite their inherent limitations. The Domain Decomposition Method (ADM) and the Homotopy Analysis Method (HAM) establish essential benchmarks for validating numerical discretizations of the initial temporal regime.

While the resulting closed-form formulations may be simplified, they yield intuitively comprehensible insights regarding parameter dependence that might otherwise be obscured in purely numerical representations. Consequently, such approaches remain central to expedited exploratory studies and educational applications. Several weaknesses warrant acknowledgment. The current study assumes spatial homogeneity of the diffusion coefficient and intrinsic growth rate, whereas empirical ecosystems exhibit significant spatial heterogeneity. The FV-L1 framework would necessitate a more refined approach to handling discontinuities during flux reconstruction to accommodate spatially variable coefficients.

Moreover, the incorporation of stochastic factors, which reflect environmental variability or demographic stochasticity, is essential for enhancing ecological realism. In practical applications, specific research objectives will dictate the precise purposes of the numerical method. In straightforward simulations over short timeframes with simple geometries, the FD-L1 scheme offers a favorable balance between accuracy and computational simplicity.

Conversely, when the preservation of mass, nutrient cycling, or other system properties is critical such as in closed systems or mass-balanced ecosystems the FV-L1 approach emerges as the superior choice, notwithstanding the challenges associated with its implementation. It is advisable to regard ADM and HAM primarily as validation and theoretical frameworks, rather than as computational engines. The implications of these findings extend beyond the Fisher-KPP framework, as many ecological models share similar mathematical structures characterized by well-dispersed diffusion and nonlinear population dynamics. The conservative discretization theory outlined herein can be applied to Lotka-Volterra systems, epidemic models, and spatial ecology models. Given the increasing focus of computational ecology on global change scenarios that require long-term projections, the.

7. Discussion: Ecological Implications and Methodological Recommendations

The fractional order α is a quantitative measure of the permeability of the landscape. Field-derived values are the determination of the observed invasion velocities or dispersal kernels, which provide a direct understanding of the connectivity of the habitat without explicit spatial mapping of barriers. In numerical experiments, we demonstrate that 1.0 or 0.6 is a more effective way of decreasing the speed of wave fronts by about 60%, which is a significant amount and consistent with empirical studies of species invasion between fragmented and continuous landscapes. Mass conservation is one of the most important numerical considerations of ecological applications. Many population models use no-flux boundary conditions, indicating closed systems with equal total biomass. The contrived loss of mass in the FD-L1 scheme, though small in absolute terms, systematically increases over timelike periods of the ecological forecasting (years to decades). This systematic bias can prejudice long-term forecasts of population sustainability, especially in endangered species whose precise abundance determination is crucial. The scheme can also be made more efficient in large-scale ecological simulations by integration of the advanced computational schemes such as parametric surface representations [16].

The finite-volume method eliminates the above limitation due to its mathematical nature. The fact that the conservation laws are discretized as opposed to the differential equations conserves the nature of the intrinsic balance between the influx-outflow and internal reaction processes. This property acquires progressively greater importance with an increase in the spatial dimensionality of the model or with the addition of more complex reaction kinetics. Despite the additional implementation complexity, which would require a careful flux reconstruction and cell-averaging process, the ultimate improvement in numerical reliability is worthwhile.

The methods of analysis have relevance, despite the limitations inherent in them. The ADM and HAM provide much-needed benchmarks for the validation of numerical discretizations of the initial temporal regime. Although the resulting closed-form formulae are taglines, they provide intuitively understandable information about the dependence on parameters, which might otherwise be hidden within purely numerical representations. Accordingly, such approaches are still central in expedited exploratory studies and instructional uses. There are several weaknesses that should be noted. The current study presupposes the spatial homogeneity of the diffusion coefficient ‛D and the intrinsic growth rate ‛r, but empirical ecosystems are characterized by strong spatial heterogeneity. The FV-L1 framework would require a finer handling of discontinuities occurring during flux reconstruction to be extended to spatially variable coefficients.
Furthermore, the addition of stochastic factors, which are characteristic of environmental variability or demographic stochasticity, is a mandatory addition to ecological realism. In the field, the specific research objectives depend on the exact purposes of the numerical method. In simple simulations of a short period with simple geometries, the FD-L1 scheme provides a good tradeoff between accuracy and simplicity of computation.
Conversely, when the preservation of mass, nutrient cycling, or other properties of the system are obligatory, such as in closed systems and mass-balanced ecosystems, the FV-L1 approach would be the best choice despite the difficulties related to its application. ADM and HAM should be more like validation and theoretical than computing engines. These results have broader implications than the Fisher-Kpp framework. Many ecological models have similar mathematical forms, with well-dispersed diffusion and nonlinear population dynamics. The conservative discretization theory demonstrated below can be applied to Lotka-Volterra systems, epidemic models, and spatial ecology models. With an increasing amount of computational ecology addressing global change situations where projections over long time horizons are required, the dominance of numerical reliability cannot be determined.

8. Conclusion and Future Directions

This paper presents a comparative systematic analysis of four solution methodologies of time-fractional Fisher-KPP equations, namely: the ADM and HAM, the finite-difference L1 scheme (FD-L1), and the finite-volume L1 scheme (FV-L1). In a fractional-calculus model, the memory effects of subdiffusive dispersal in fragmented habitats are effectively described. Numerical experiments reveal trade-offs that inherently exist between accuracy, conservation, and computational efficiency. The FV-L1 discretization is better in terms of mass conservation properties as it preserves the total population mass to machine precision over longer simulations. The ecological uses of this fidelity cannot be avoided, since the use of artificial numerical dissipation may lead to a biased estimate of population viability. Despite the operation requiring painstaking care to recreate the flux, the resulting consistency warrants extra effort in models in need of very stringent conservation.

Various opportunities for further work are revealed. An urgent need is the generalization of the FV1 system to two and three space dimensions, thus allowing the simulation of the invasive species spread on heterogeneous terrains. Additional ecological realism would be achieved by introducing stochastic elements, i.e., parametric uncertainty and additive noise.

In addition to this, optimal utilization of computational efficiency could be achieved by developing adaptive strategies that dynamically choose between analytical and numerical strategies depending on the properties of solutions. Theoretically, the interrelations between fractional order a and quantifiable landscape measures are worth investigating. Empirical data associated with indices of habitat fragmentation have the potential to transform the parameter into a biologically relevant quantity. Empirical grounding of this nature would strengthen the connection between mathematical theory and conservation practice. The methodology offered here has applicability outside of ecological modeling. The conservative discretization method, as illustrated here, could be useful in any physical or biological system that obeys conservation laws with memory effects, such as contaminant transport in porous media, calcium dynamics in neuronal dendrites, or the like. The concepts of mass-conserving numerical schemes are general. The FV-L1 framework will be expanded to two and three spatial dimensions in future research, as was done with more established PDE modeling methods to complex geometries [14], especially to the spread of invasive species on a heterogeneous landscape.

In this way, this work can be relevant to both computational mathematics and theoretical ecology. It provides practitioners with clear guidelines on how to choose methods depending on their own needs and provides theorists with a sound numerical framework to study fractional dynamics in complex systems. With ecological forecasting becoming increasingly confronted with conservation issues of immediate concern, the dependability of computing means goes beyond mere convenience to necessity.

Conflict of Interest

The author declares no conflicts of interest regarding the publication of this article.

Data Availability

The data generated and analyzed during this research are purely synthetic and generated from the time fractional Fisher KPP model that has been described in Sections 2 and 3 of this work. There are no independent datasets used within this research. All results obtained are replicable by using the numerical schemes FD-L1 and FV-L1 mentioned within this manuscript. A MATLAB/Python code for the numerical schemes, along with parameter files for generating plots and tables, will be made publicly available after acceptance of this manuscript via a reputable data deposition platform like GitHub or Zenodo as per journal policy.

9. Acknowledgments

This research received no specific grant from funding agencies in the public, commercial, or not-for-profit sectors. The author thanks the three anonymous reviewers for constructive feedback, which significantly improved the manuscript.

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